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A Model of Sam Loyd's Outwitting the Weighing Machine

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Abstract

Alternative solution methods for the puzzle Outwitting the Weighing Machine by Sam Loyd are considered. The puzzle provides a great opportunity to study how Excel Solver and LINGO can be used to find solutions to challenging problems.

Key words: Optimization, Excel, Spreadsheet, Lingo, Solver

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INTRODUCTION

The wide-spread availability and popularity of Sudoku puzzles has revealed new and renewed interest in various other forms of mathematical puzzles and riddles, known as recreational mathematics. Aside: There are several of the reasons for this popularity; details are discussed in Friesen, et. al (2013). Some of the better known writers in the field include H. E. Dudeney, Sam Loyd, Raymond Smullyan, Martin Gardner and Charles Lutwidge Dodgson, better known as Lewis Carrol, author of *Alice's Adventures in Wonderland*.

The purpose of this paper is to illustrate how to build a mathematical model to solve Sam Loyd's puzzle Outwitting the Weighing Machine. As discussed in the Literature Review below, students often benefit by using recreational mathematics in mastering logic and programming concepts.

LITERATURE REVIEW

American Sam Loyd (1841-1911) was a chess player, puzzle author and recreational mathematician; furthermore, he was extremely prolific. Perhaps his best known work is the self-published *Sam Loyd's Cyclopedia of 5000 Puzzles Tricks and Conundrums with Answers* (1914) that was published after his death. He is one of the best known writers in the field of recreational mathematics.

The best known peer-reviewed journal devoted to recreational mathematics is the *Journal of Recreational Mathematics* (Baywood Publishing, Inc., n.d.). Many journals give some passing attention to the subject, however, often through dedicated columns. For example, *Communications of the ACM* regularly publishes a column named "last byte" (Winkler, 2012). Alexander Dewdney (not to be confused with H. E. Dudeney!) wrote a "famous section" in *Scientific American* during the 1980s, as did Gardner for over twenty-four years prior. Dewdney's column was named "Computer Recreations" while Gardener's column was named "Mathematical Games" (Jimenez & Munoz, 2011). The column "Classroom Capsules" appears in *The College Mathematics Journal.* Problems and puzzles often appear in the column to provide an "effective teaching strategy or tool" for college mathematics instruction (Alfaro, Han, & Schilling 2011). In the early twentieth century, Carver (1923) suggested using Dudeney's puzzles as "stimulus" to undertake investigation, for both students and teachers! In an empirical study on game-based learning, Hamari and others (2016) found that engagement with the game improves learning, as does achieving an acceptably high level of challenge. Probably outgrowths of recreational mathematics, Jimenez and Munoz (2011) describe "recreational programming," while Demain (2010) uses the term "recreational computer science." In the next section, the subject instance of recreational mathematics is defined.

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Sam Loyd's Outwitting the Weighing Machine (1914)
The puzzle is re-printed below:

The Puzzle: The five school children in couples weigh 129 pounds, 125 pounds, 124 pounds, 123 pounds, 122 pounds, 121 pounds, 120 pound, 118 pounds, 116 pounds and 114 pounds on a weighing machine. What was the weight of each one of the five little girls if taken separately?

The answer to the puzzle can be found on numerous websites including the logic and answers in mathisfun.com (n.d.) and puzzles-answer.blogspot.com (2013). Other websites, like github.com (n.d.) provide computer code. As shown in the next section, one possible solution method is to frame the problem as a mathematical model.

Outwitting the Weighing Machine as a Mathematical Model
The formulation of the puzzle as a mathematical model is displayed below:

Minimize: a
Subject to:
a<b<c<d<e
a,b,c,d,e = integer
a+b=114
a+c=116
e+d=129
e+c=125
all=a+b+c+d+e
abde=a+b+d+e
c=all-abde
a+d<=124
a+e<=124
b+c<=124
b+d<=124
b+e<=124
c+d<=124
a+d>=120
a+e>=120
b+c>=120
b+d>=120
b+e>=120
c+d>=120

where:
a=smallest weight
b=second smallest weigh
c=median weigh
d=second heaviest weight
e=heaviest weight

The next section demonstrates an alternative formulation, this time in Excel.

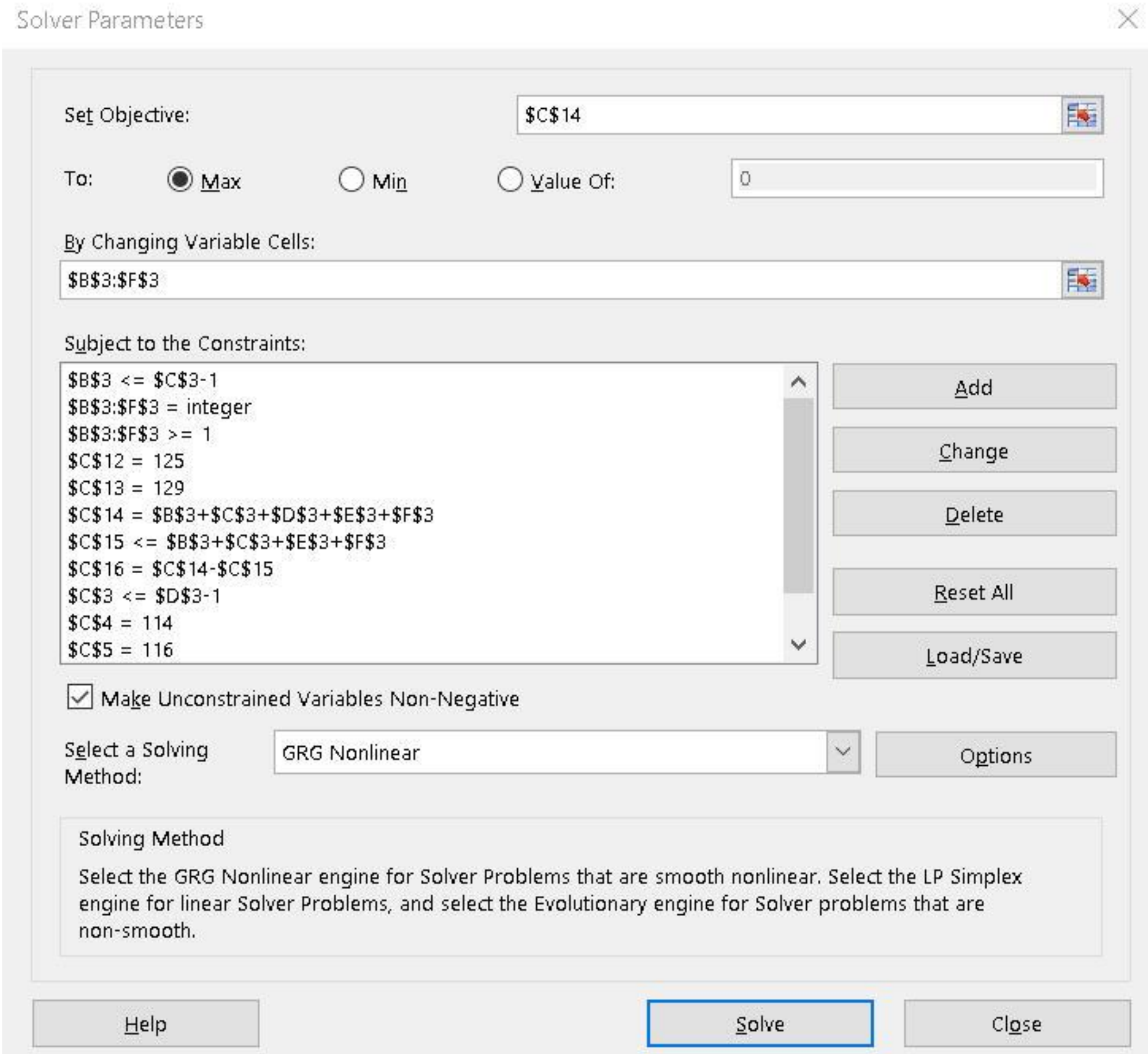
An Excel Spreadsheet Model of Outwitting the Weighing Machine

The Weighing Machine puzzle can also be formulated and solved using the Excel Solver. Solver is a software tool developed by Frontline Systems (Solver.com 2017) as an add-in for Microsoft Excel. The initial Solver formulation is presented in Table 1. The Solver model parameters are displayed in Figure 1. The formula view of the spreadsheet is shown in Table 2. The spreadsheet solution is presented in Table 3. The solution states that the weights of the young girls are 56, 58, 60, 64 and 65. All of the constraints are satisfied. Solver Options presents a check box labeled "Ignore Integer Constraints"; this must be unchecked. Otherwise you may not receive the optimal solution.

Table 1
Excel Solver Spreadsheet Model Formulation of Outwitting the Weighing Machine

1/A	B	C	D	E	F
2	A	B	C	D	E
3	0	0	0	0	0
4	A+B	0			
5	A+C	0			
6	A+D	0			
7	A+E	0			
8	B+C	0			
9	B+D	0			
10	B+E	0			
11	C+D	0			
12	C+E	0			
13	D+E	0			
14	ALL	0			
15	A+B+D+E	0			
16	ALL-(A+B+D+E)	0			

Figure 1
Solver Parameters for Outwitting the Weighing Machine Puzzle



There are two additional constraints that are not displayed in Figure 1. These two constraints are displayed below:
$D$3<=$E$3-1
$E$3<=$F$3-1

Table 2
Excel Solver Spreadsheet Formula View for Outwitting the Weighing Machine

1/A	B	C	D	E	F
2	A	B	C	D	E
3	0	0	0	0	0
4	A+B	=B3+C3			
5	A+C	=B3+D3			
6	A+D	=B3+E3			
7	A+E	=B3+F3			
8	B+C	=C3+D3			
9	B+D	=C3+E3			
10	B+E	=C3+F3			
11	C+D	=D3+E3			
12	C+E	=D3+F3			
13	D+E	=E3+F3			
14	ALL	=B3+C3+D3+E3+F3			
15	A+B+D+E	=B3+C3+E3+F3			
16	ALL-(A+B+D+E)	=C14-C15			

Table 3
Excel Solver Solution for Outwitting the Weighing Machine

1/A	B	C	D	E	F
2	A	B	C	D	E
3	56	58	60	64	65
4	A+B	114			
5	A+C	116			
6	A+D	120			
7	A+E	121			
8	B+C	118			
9	B+D	122			
10	B+E	123			
11	C+D	124			
12	C+E	125			
13	D+E	129			
14	ALL	303			
15	A+B+D+E	243			
16	ALL-(A+B+D+E)	60			

A LINGO Model for Outwitting the Weighing Machine

An alternative approach to solving the Weighing Machine puzzle is presented using the popular modeling language LINGO. The software is a product of LINDO Systems, (LINDO SYSTEMS, 2017), the developer of other widely-used optimization software including LINDO and What's Best! The LINGO formulation is displayed below and illustrates how close the software formulation is to the general mathematical model formulation discussed previously.

```
Model:
Min =all;
a<=b-1;
b<=c-1;
c<=d+1;
d<=e+1;
a>=1;
b>=1;
c>=1;
d>=1;
e>=1;
d+e=129;
c+e=125;
a+b=114;
a+c=116;
all=a+b+c+d+e;
abde=a+b+d+e;
c=all-abde;
@gin (a);
@gin (b);
@gin (c);
@gin (d);
@gin (e);
END
```

The LINGO output from the model is displayed in Figure 2. Again, the solution weights are 56, 58, 60, 64 and 65 pounds.

Figure 2: LINGO Model Output for Outwitting the Weighing Machine

Global optimal solution found.

Objective value:	303.0000
Objective bound:	303.0000
Infeasibilities:	0.000000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
ALL	303.0000	0.000000
A	56.00000	1.000000
B	58.00000	1.000000
C	60.00000	1.000000
D	64.00000	1.000000
E	65.00000	1.000000
ABDE	243.0000	0.000000

Row	Slack or Surplus	Dual Price
1	303.0000	-1.000000
2	1.000000	0.000000
3	1.000000	0.000000
4	5.000000	0.000000
5	2.000000	0.000000
6	55.00000	0.000000
7	57.00000	0.000000
8	59.00000	0.000000
9	63.00000	0.000000
10	64.00000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	-1.000000
16	0.000000	0.000000
17	0.000000	0.000000

Summary

Puzzles and riddles, such as Sam Loyd's Outwitting the Weighing Machine, provide exercises in logic and mathematical reasoning. The puzzle, written some 100 years ago, was undoubtedly designed as a pencil-and-paper exercise. However, it can be modeled and solved using readily available software, such as Excel Solver and LINGO. The software models in this paper illustrate that Outwitting the Weighing Machine would provide the basis for a challenging learning exercise for any operations research-oriented class.

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