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Voting or technical evaluation: the optimal rule to choose a manager under uncertainty

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Abstract
The choice of managers is one of the most important tasks in any firm. This choice is often difficult because the firm does not know perfectly the ability of candidates to perform the job. To deal with this uncertainty, it is possible to choose managers through a voting procedure in a board, or one may use some exogenous technical evaluation, such as an exam or a contest, to try to learn about the quality of each candidate. There is a trade-off between these two procedures: voting decreases the weight of idiosyncratic errors, but is subject to a public good problem that affects any electoral procedure since the effort by one member to learn about the candidates benefits all other members through an increased probability of choosing correctly. The other members may free-ride on this effort and decrease the amount of time they dedicate to gathering information about the candidates. I show that a technical evaluation gives a higher probability of choosing correctly even when it is less efficient than any individual analysis: the free-riding problem is more serious than the risk of idiosyncratic errors. Voting should not be used when there is a problem of asymmetric information; it is only a tool to aggregate heterogeneous preferences.

Key words: choice of managers; voting; corporate structure

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INTRODUCTION

Delegation is a fundamental task in management. Shareholders choose board members, who choose top managers to pick lower-level managers and other employees. A central issue in management is the procedure to choose the managers responsible for the delegated tasks.

Two common methods are voting and technical evaluations. CEOs are typically chosen in a voting of the board of directors. Lower-level managers are usually chosen by some direct evaluation by the CEO, or on his behalf. The latter may either be a subjective analysis of a *Curriculum Vitae*, or some exogenous measurement. It any case, it usually combines both objective and subjective evaluations from different instances: the department of human resources, the manager who will oversee the employee's work, and sometimes the top management.

Both procedures aim at minimizing the unavoidable uncertainty in the evaluation of any given candidate: if the quality and the potential contribution of the potential employee were immediately available and comparable, then the choice would be trivial. However, this is usually not the case: either there is relevant information about the candidates unknown to the firm, or there is no natural way to rank them even when all the relevant information may be used. In the present paper, I consider the first of these issues and pose the following question: what is the best rule to choose a manager when candidates may be ranked objectively but the firm does not have all the necessary information about them?

This is a typical problem of asymmetric information, which induces some uncertainty for the firm. In principle, one might consider a contract-theory approach: the firm might be able to elicit the necessary information with some contractual device such as workload and payment - the candidate's choice of a given contract reveals his type. This sophisticated practice, however, is often limited in practice: contracts cannot be tailored to the employee and therefore must, at least to some extent, follow a standard model to anyone approved for a given position. In such cases, the firm must rely solely on the direct evaluation of potential candidates.

Voting uses individual evaluations by the members of a board. In informal terms, it takes an average of several opinions in the hope that the median voter tends to choose correctly: by doing so, it reduces the

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weight of idiosyncratic errors. However, a free-riding problem arises[2]: the effort of a given member to learn about the candidates and choose correctly benefits all other members through an increased probability of appointing the right candidate. This public good problem prevents the board from minimizing the probability of a poor choice.

An alternative is the use of some type of technical, and exogenous, evaluation. One may think of some kind of standard test, or an external evaluation. This avoids the free-riding problem but incurs in another cost: the choice becomes idiosyncratic as an eventual mistake by the external evaluation device has a lower possibility of being opposed and disputed.

I study this tradeoff theoretically with a choice model and apply it concretely through a numerical simulation to establish which method achieves the highest probability of choosing correctly. I find that, whenever there is an objective (and initially unknown) measure of the type of candidates, the technical evaluation gives the higher probability of choosing correctly. This holds *even if it is less efficient than any individual evaluation from board members* and under the mildest possible externality. In other words, the free-riding problem is typically the most relevant issue. This implies that voting is an ineffective device to capture information. It should only be used when there are strong subjective components in the choice of a manager, and no need to elicit relevant information about candidates.

Voting is studied in many fields, from political science to corporate finance. In economics, it falls under the larger theory of social choice and tries to build answers to the celebrated impossibility theorem in Arrow (1951)[3]. The study of free-riding in voting developed in a large literature: Tuck (2008) and Struthers (1989) are major reviews, and Karakaya (2009) presents some experiments as a method to test the theory empirically. Voting, whether in a board of directors or in national elections, is studied mostly as a tool to aggregate heterogeneous preferences, but less attention has been devoted to the need to acquire information, and how this compares to other choice schemes such as contests or exams. This paper aims at filling this gap.

The rest of the paper is organized as follows. Section 2 lays out the basic model. Section 3 computes the probabilities of choosing the correct candidate under voting and under the technical evaluation and compares them. Section 4 briefly concludes.

MODEL

There are two candidates, labeled A and B, to take a managing position under the board of directors. There are objective measures of quality (real numbers) for each candidate: θ_A and θ_B. For the rest of the paper, I will assume without loss of generality that candidate A is the best one: $\theta_A > \theta_B$. All members of the board would like to choose him if they knew θ_A and θ_B, but each candidate is privately informed about his type. I will also assume that the contract offered to the winner is the same irrespective of the type; this implies that it is not possible to use it a screening device[4].

The board may use two different procedures to choose one of the candidates. The first is an exogenous technical evaluation of each of them. One may interpret it as some type of contest, or as an exam or interview that both candidates must take. The one who achieves the highest score in the exam gets the position. The second procedure is a simple voting scheme: the candidate with most votes in the board is chosen.

Both methods are subject to uncertainty. The technical evaluation may assign the highest score to the worst candidate, while the board may vote poorly. Both methods have advantages and disadvantages to deal with this problem. One the one hand, voting decreases the probability that one sole idiosyncratic error will give the position to the bad candidate. On the other hand, it generates a free-riding problem that induces every voter to work less, and therefore the signal becomes noisier.

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[2] *For a complete discussion, see Mas Colell et al. (1995), chapter 11.*
[3] *For a recent review of all developments that followed the seminal paper, see Kelly (2004).*
[4] *This is the case, for example, if output measures are very noisy.*

The next section determines the probability of choosing the best candidate under each procedure and compares them[5].

RESULTS

Technical Evaluation

Consider first that the candidates A and B are compared according to some objective measure, which cannot be disputed by the members of the board, who decide previously on the method to compare the candidates. As mentioned above, this may be seen as an objective exam that both candidates must take. The one who achieves the highest result gets the position.
The aim of this objective measure is to capture the values θ_A and θ_B. If it were a perfect measure, then it would always pick the best candidate since $\theta_A > \theta_B$. There is however some amount of uncertainty. The grade of each candidate is $g_A = \theta_A + \varepsilon_A$ and $g_B = \theta_B + \varepsilon_B$, in which ε_A and ε_B are i.i.d random variables with uniform distributions $U[- \varepsilon , \varepsilon]$ for some strictly positive ε.
One may interpret this error as an unavoidable imprecision that plagues any objective measure. In the case of an exam or an interview, one candidate may have a particularly good or bad day, biasing the evaluation.
The best candidate wins if and only if $g_A \geq g_B$. This happens with probability:

$$Prob(g_A \geq g_B / \varepsilon) = Prob(\theta_A + \varepsilon_A \geq \theta_B + \varepsilon_B / \varepsilon) = Prob(\varepsilon_B - \varepsilon_A \leq \theta_A - \theta_B / \varepsilon)$$

Let $Z= \varepsilon_B - \varepsilon_A$ label the difference between these two random variables. Using the fact that $(\theta_A - \theta_B) > 0$, the distribution of Z may be written is as follows.

$$F(\theta_A - \theta_B) =$$
$$Prob(Z \leq \theta_A - \theta_B) = \begin{cases} [- (\theta_A - \theta_B)^2 +4(\theta_A - \theta_B) \varepsilon + 4\varepsilon^2] / 8\varepsilon^2 , & \text{if } 0 \leq (\theta_A - \theta_B) < 2\varepsilon \\ 1 , & \text{if } 2\varepsilon \leq (\theta_A - \theta_B) \end{cases}$$

If $\varepsilon = 0$, then $(\theta_A - \theta_B) > 0$ implies that the signals g_A and g_B are perfectly informative and the best candidate wins with probability one. More generally, for $\varepsilon > 0$, this depends on the sign of $2\varepsilon - (\theta_A - \theta_B)$: if it is negative, then again the best candidate wins with probability one. In either case, the technical evaluation is trivially the best option. Turning now to the case $(\theta_A - \theta_B) < 2\varepsilon$, one has the following:

Prob [candidate A wins/technical evaluation] =

$$Prob(g_A \geq g_B / \varepsilon) = [- (\theta_A - \theta_B)^2 +4(\theta_A - \theta_B)\varepsilon + 4\varepsilon^2] / 8\varepsilon^2$$

Voting

Each voter has an individual evaluation θ_{ji} for *j=A,B*. This is random variable drawn from an uniform distribution $U[\theta_j - \varepsilon_i , \theta_j + \varepsilon_i]$, in which ϵ_i is an error term chosen by voter *i* at some personal cost. It captures how much effort he puts into the evaluation of each candidate. The density function for this distribution is:

$$f(\theta_{ji} / \epsilon_i) = \frac{1}{2\epsilon_i}$$

Notice that this does not depend on *j* since the error term ϵ_i is the same for both candidates; one may interpret this as a relative error. Assuming that the random variables are independent, given the support of each one, the joint distribution has the following density:

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[5] *The probability of picking the best candidate is not necessarily the objective function of the board members. This is the case, for example, if they are risk-neutral and the benefit from each candidate is constant, exogenous, and normalized suitably.*

$f(\theta_{Ai}, \theta_{Bi} / \epsilon_i) = \frac{1}{4{\epsilon_i}^2}$

If the voter makes the error equal to zero, then the distribution becomes degenerate and $\theta_{ji} = \theta_j$ with certainty. There is however a cost when minimizing ϵ_i (modeled below).

Given ϵ_i, one may compute the probability that voter *i* will choose the right candidate as follows.

Prob[*i* chooses correctly] =
$\text{Prob}[\theta_{Ai} \geq \theta_{Bi}]$=

$$\left[\int_{(\theta_A-\epsilon_i)}^{(\theta_A+\epsilon_i)} \int_{(\theta_B-\epsilon_i)}^{(\theta_{Ai})} \frac{1}{4{\epsilon_i}^2} d\tilde{\theta}_{Bi}\, d\tilde{\theta}_{Ai}\right]$$

In which tildes indicate dummy integration variables. One may rewrite this as follows:

$$\frac{1}{4{\epsilon_i}^2}\left[\int_{(\theta_A-\epsilon_i)}^{(\theta_A+\epsilon_i)} \left(\int_{(\theta_B-\epsilon_i)}^{(\theta_{Ai})} d\tilde{\theta}_{Bi}\right) d\tilde{\theta}_{Ai}\right] =$$

$$\frac{1}{4{\epsilon_i}^2}\left[\left(\frac{(\theta_A+\epsilon_i)^2}{2} - (\theta_B-\epsilon_i)(\theta_A+\epsilon_i)\right) - \left(\frac{(\theta_A-\epsilon_i)^2}{2} - (\theta_B-\epsilon_i)(\theta_A-\epsilon_i)\right)\right]$$

Simplifying this last expression, one gets:

$$\frac{1}{2{\epsilon_i}^2}[{\epsilon_i}^2 + (\theta_A - \theta_B)\epsilon_i] =$$

$$\left[\frac{1}{2} + \frac{(\theta_A - \theta_B)}{2\epsilon i}\right]$$

For ease of notation, label this probability p_i. For future reference, compute $(1- p_i)$:

$$\left[\frac{1}{2} - \frac{(\theta_A - \theta_B)}{2\epsilon i}\right]$$

Notice that the higher the difference $(\theta_A - \theta_B)$, and the smaller the error term ϵ_i, the more likely it becomes that the voter will choose correctly: p_i is tilted above $\frac{1}{2}$. To guarantee that this expression defines a probability, assume that $\frac{(\theta A - \theta B)}{\epsilon i} \leq 1$ for all values of θ_A, θ_B and ϵ_i, or $(\theta_A - \theta_B) \leq \epsilon_i$. Again, one may interpret that the error term is relative to the difference $(\theta_A - \theta_B)$. When $\epsilon_i = (\theta_A - \theta_B)$, voter *i* chooses the right candidate with probability 1. As the error term increases or the difference $(\theta_A - \theta_B)$ decreases, this probability decreases.

For a given voter, what matters is not the probability that he will vote correctly, but the probability that the right candidate will be chosen. There are two excluding situations in which this may happen. First, voter *i* may choose the wrong candidate, but at least $\frac{n+1}{2}$ other voters will choose correctly. Second, voter i may choose correctly, and at least $\frac{n-1}{2}$ do the same. Hence the probability that the right candidate will be chosen is the following (let *B(x,y)* denote the binomial distribution with x repetitions for y successes and p_{-i} denote the probability that other players will choose candidate A).

Prob[the right candidate wins/ ϵ_i]=

$$p_i\left[\sum_{j=\frac{n-1}{2}}^{n-1} B(n-1,j)[(p_{-i})^j(1-p_{-i})^{n-1-j}]\right]+(1-p_i)\left[\sum_{j=\frac{n+1}{2}}^{n-1} B(n-1,j)p_{-i}{}^j(1-p_{-i})^{n-1-j}\right]$$

This expression boils down to the following one.

$$p_i\left[B\left(n-1,\frac{n-1}{2}\right)[(p_{-i})(1-p_{-i})]^{\frac{n-1}{2}}\right]+\left[\sum_{j=\frac{n+1}{2}}^{n-1} B(n-1,j)p_{-i}{}^j(1-p_{-i})^{n-1-j}\right]$$

The voter wants to maximize this expression but is subject to a cost when he decreases the error term ϵ_i. This cost is interpreted as his effort to learn about the candidate so as to decrease the variance in the signal θ_{ji} he gets. I model this as a benefit function $g(\epsilon_i)$ that is increasing and concave ϵ_i: the higher the error term (i.e., the lower the effort to learn), the higher the benefit, with decreasing returns. Importantly, the function g is the same for all voters. The voter chooses ϵ_i to maximize the objective function:

$$\left[\frac{1}{2}+\frac{(\theta_A-\theta_B)}{2\epsilon_i}\right]\left[B\left(n-1,\frac{n-1}{2}\right)[(p_{-i})(1-p_{-i})]^{\frac{n-1}{2}}\right]+\left[\sum_{j=\frac{n+1}{2}}^{n-1} B(n-1,j)p_{-i}{}^j(1-p_{-i})^{n-1-j}\right]+g(\epsilon_i)$$

The first-order condition for this problem is:

$$B\left(n-1,\frac{n-1}{2}\right)[(p_{-i})(1-p_{-i})]^{\frac{n-1}{2}}=\frac{-2\epsilon^2}{(\theta_A-\theta_B)}g'(\epsilon i)$$

Assuming for simplification that g has constant unitary slope, one gets:

$$B\left(n-1,\frac{n-1}{2}\right)[(p_{-i})(1-p_{-i})]^{\frac{n-1}{2}}=\frac{-2\epsilon^2}{(\theta_A-\theta_B)}$$

Since g is the same for all voters, the solution ϵ_i is the same for all *i*. Label this ϵ. Using the expressions for p_i and (1- p_i) , this becomes:

$$B\left(n-1,\frac{n-1}{2}\right)\left[\left(\frac{1}{2}+\frac{(\theta_A-\theta_B)}{2\epsilon}\right)\left(\frac{1}{2}-\frac{(\theta_A-\theta_B)}{2\epsilon}\right)\right]^{\frac{n-1}{2}}=\frac{-2\epsilon^2}{(\theta_A-\theta_B)}$$

This boils down to:

$$B\left(n-1,\frac{n-1}{2}\right)\left[\frac{1}{4}-\frac{(\theta_A-\theta_B)^2}{4\epsilon^2}\right]^{\frac{n-1}{2}}=\frac{-2\epsilon^2}{(\theta_A-\theta_B)}$$

This equation determines ϵ implicitly as a function of *n* and (θ_A - θ_B). One may use it to study how the probability that the right candidate wins is affected by these parameters and hence how it compares to the direct-competition method.

The role of (θ_A - θ_B)

To illustrate the role of the difference (θ_A - θ_B), make n=3 in the expression above. This is the lowest odd number of participants that allows for a non-trivial voting procedure. Hence I rule away any spurious effect that might come from an extreme problem of free-riding. In other words, I minimize the impact of free-riding on the solution to study the impact of (θ_A - θ_B). The expression above becomes:

$$4\epsilon^4 + \epsilon^2(\theta_A - \theta_B) - (\theta_A - \theta_B)^3 = 0$$

If $\epsilon = (\theta_A - \theta_B)$, then as noted above the probability of choosing the right candidate is equal to one. In this case, the equation above becomes:

$$4\epsilon^4 + \epsilon^3 - \epsilon^3 = 0$$
$$4\epsilon^4 = 0$$
$$\epsilon = 0$$

That is, the error term is equal to zero. However, one may see that this is not a solution if $\epsilon < (\theta_A - \theta_B)$ since $\theta_A \neq \theta_B$. For $\epsilon < (\theta_A - \theta_B)$, one may solve for ϵ using ϵ^2 as the variable of a quadratic equation. Discarding the negative root, and defining for ease of notation $d = \theta_A - \theta_B$, one has:

$$\epsilon^2 = \frac{-d+\sqrt{d^2+16d}}{8}$$

Discarding again the negative root, the following expression is obtained:

$$\epsilon = \sqrt{\frac{-d+\sqrt{d^2+16d}}{8}}$$

Define p= p(ϵ). The probability that the right candidate wins is:

Prob[candidate A wins/voting] =

$$B\left(n, \frac{n+1}{2}\right) p^n (1-p)^{\frac{n-1}{2}} =$$

$$B\left(n, \frac{n+1}{2}\right) \left[\frac{1}{2} + \frac{d}{2\epsilon}\right]^n \left[\frac{1}{2} - \frac{d}{2\epsilon}\right]^{\frac{n-1}{2}} =$$

$$B(3,2)\left[\frac{1}{2} + \frac{d}{2\sqrt{\frac{-d+\sqrt{d^2+16d}}{8}}}\right]^3 \left[\frac{1}{2} - \frac{d}{2\sqrt{\frac{-d+\sqrt{d^2+16d}}{8}}}\right]^2$$

This determines a well-defined probability p since $d \equiv (\theta_A - \theta_B) \leq \epsilon$: this expression defines a strictly positive value p_i [6]. More importantly, it implies that p_i is strictly increasing in the difference $d \equiv (\theta_A - \theta_B)$. Intuitively, when the difference between the candidates increases, it becomes more likely that the voting procedure chooses correctly.

Strikingly, this probability is much lower than 1 even for n=3 , when the problem of free-riding is as small as possible. This is bounded above by 0.6 for all values of $d \equiv (\theta_A - \theta_B)$ smaller than ϵ in the solution. As expected, this upper bound decreases as *n* increases: when the number of voters increase, the free-riding

[6] *One may disregard any solutions such that $e > \theta A - \theta B$. This amounts to imposing an adequate upper bound on the difference $d \equiv (\theta_A - \theta_B)$.*

problem is worsened. For reference, the equilibrium value of ϵ is also bounded above by 0.7 in the same range.

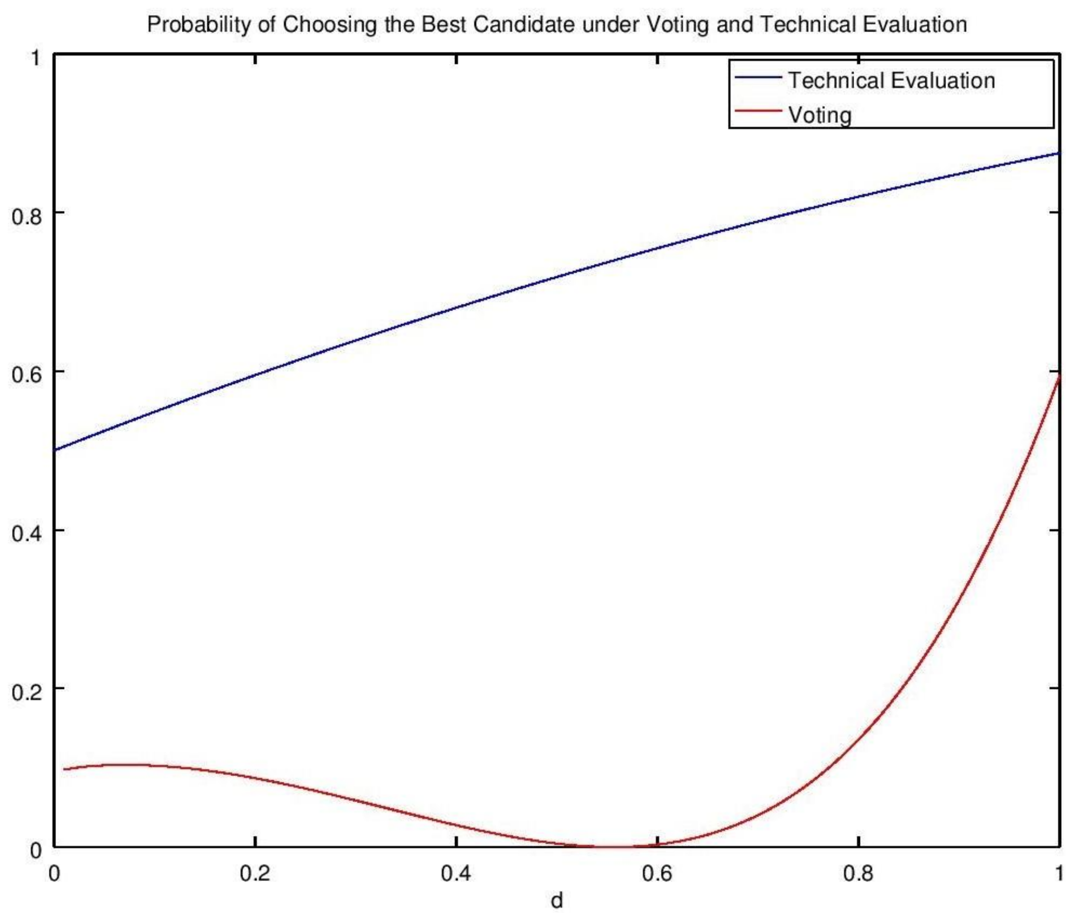
COMPARISON

I turn now to the comparison of the expressions found above for:

Prob[candidate A wins/technical evaluation]

Prob[candidate A wins /voting]

I will assume that technical evaluation is less efficient than any individual evaluation. This will avoid any ex-ante bias that might favor it over voting. Specifically, I will take $\varepsilon = 1$, higher than the upper bound 0.7 of the error term in the individual evaluation. Moreover, I will keep the assumption that n=3. This setup favors voting over the technical evaluation. Still, the probability that the technical evaluation will choose correctly ranges from 0.5 (when $d \equiv (\theta_A - \theta_B)$ is close to the lower bound, which is zero) to one as it increases. In fact, it is strictly higher under than under voting for any value $d \equiv (\theta_A - \theta_B)$, as one may see in the following graphic.



This discussion is summarized in the proposition below.

Proposition. The probability of choosing the right candidate is strictly higher under the technical evaluation than under voting. Moreover, it has following features:
- It has an upper bound strictly lower than one under voting, while it converges to one under the technical evaluation as $d \equiv (\theta_A - \theta_B)$ increases.
- The lower bound is 0.5 for the technical evaluation and below this for voting.
- It is strictly increasing in $d \equiv (\theta_A - \theta_B)$ under the technical evaluation but non-monotonic under voting.

One may conclude that voting is, in general, not an efficient tool to choose a manager for an enterprise when there is an objective measure of managerial quality that may be evaluated, even imprecisely, with some external tool. This comes from the fact that the well-known free-riding problem that affects any electoral procedure is worse than the imperfectness of a given measurement. As a corollary, voting should be used when there are ideological differences that do not allow candidates to be compared according to an objective measure.

CONCLUDING REMARKS

This paper discussed the optimal choice of a manager when there is uncertainty about the candidates. The main finding is that voting by board members typically generates a low probability of choosing the correct one due to a free-riding problem. Technical evaluations, on the other hand, achieve a higher probability even if they are subject to idiosyncratic errors that are not averaged out by other opinions.

The present analysis should be extended for other distributions for errors, and for general cost functions for voting. This would allow one to use firm-level data to compute actual choice probabilities in specific applications.

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