
An In-house Production or Outsourcing Decision Model based on a Stochastic Programming Approach

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Abstract

As the production capacity of a company over a certain period of time is limited, enterprises must carefully consider product line development or outsourcing options. Unlike traditional studies that use static or comparative static analyses to determine optimal production strategies, this paper proposes a stochastic programming model that can be used to determine optimum quantities of multiphase development or outsourcing. The proposed model can be used as a reference guideline for future production allocation decisions made in high-tech industries that face uncertain demands. It can also be used as a basis for future financial projections.

Key words: Production decision, Outsourcing, Stochastic models, Time series models, Stochastic programming



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INTRODUCTION

Ever since the Industrial Revolution, production manufacturing constituted an important subject of management theory and practice, with the earliest investigations focusing on ways to maximize production capacities or minimize costs of in-house production. This initial decision-making model laid the foundations for production decision-making studies that did not seriously consider supply and demand statuses or assume that demands are greater than supplies (namely, by “producer orientation” and not by “demand orientation”). Since the 1970s, however, as international market demands have become increasingly volatile and as the international production system moves towards the division of labor and specialization, production management research continues to expand. Outsourcing (production outsourcing) constitutes a major important shift characteristic of these accelerating trends.

Though outsourcing has been in practical operation for a long time, it only became well known after Prahalad and Hamel (1990) proposed the concept of corporate core competence. The purpose of outsourcing is to allow a company to subcontract non-core, auxiliary functions or operations to external specialized firms through the signing of business contracts to use these firms’ expertise and strengths to improve the overall firm efficiency and competence. Via outsourcing, a company can not only reduce operating costs, focusing resources on the development of core strengths, on the fulfillment of customer demands, and on increasing competitiveness in the market. Rather, a firm can also fully use external resources to compensate for shortcomings in its own capabilities. Meanwhile, outsourcing can also allow a company to maintain management and business flexibility and diversity. The latter is especially instrumental in allowing a company to adapt to today's ever-unpredictable business demands (Kremic, Tukel, and Rom, 2006; Moon, 2010; Razaque and Sheng, 1998; Saouma, 2008).

Courtney, Kirkland and Viguerie (1997) identified four levels of environmental uncertainty (“a clear-enough future,” “alternate futures,” “a range of futures,” and

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“true ambiguity”) and corresponding decision-making techniques. Information that can be obtained at Level One is relatively adequate and comprehensive and uncertainties are at the lowest levels, and thus, the manager can use traditional analytical tools (for example: market research, five forces analysis, value chain analysis) to determine tactical strategies. At Level Two, it is understood that several possible outcomes may result, but there is no way to identify which condition will occur in the future. Therefore, various value models are designed for each possibility, and the decision analysis framework is used to assess the risks and benefits of different plans. Level Three uses several key variables to determine feasible future ranges, though the feasible range is too wide and tends require more comprehensive definition for tactical strategies to be determined. Aside from analysis tools used for Level Two conditions, there is also a need to plan for each scenario to supplement demand forecasts and analyses. Finally, as Level Four is characterized by mutual effects of numerous sources of uncertainty—creating the most ambiguous and unpredictable environment of all the levels—and presents the highest levels of uncertainty, system simulation methods must be employed to simulate possible decision schemes.

From 2000 onwards, the high-tech industry’s global supply chain system has entered Level Four’s “true ambiguity” phase. Over the past five years, market demands have become more unpredictable, and product life cycles have shortened rapidly. These circumstances have driven the high-tech industry to cite more rigorous mathematical models to attempt to address the challenges of a highly uncertain environment. The purpose of this research paper is to employ a system stochastic simulation technique to optimally solve the high-tech industry’s optimal allocation ratio for in-house production or outsourcing and to further use sensitivity analysis and scenario simulation methods to examine effects of changes in each random variable on enterprise profit maximization.

The remainder of the paper is organized as follows. Section Two presents a review of the literature on production decision-making. Section Three describes stochastic programming specifications of the optimal in-house production and outsourcing method proposed in this study. Conclusions and recommendations are presented at the end of the paper.

LITERATURE REVIEW

Traditional economics classifies the market and further investigates decision-making behaviors produced in different markets. The market is typically separated into two main categories: a complete competitive market and an incomplete competitive market. The latter can be subdivided into an oligopolistic market and monopolistic market. As complete competitive and monopolistic markets belong to the two extremes of the market, fewer subjects can be examined, and thus, only production behaviors found in an oligopolistic market garner special concern.

Oligopolistic markets include a small number of companies, so few that each company’s decisions have an effect on the other companies. The three main oligopolistic enterprise competition model include the Cournot, Bertrand, and Stackelberg models. These models typically assume that the market includes two representative companies. Namely, by first simplifying the market as a duopoly to investigate competition and production decisions, models can then be applied to study several companies. The Cournot model assumes that the duopolistic market includes two companies of equal status that produce homogeneous products. Both parties face the same demand curve. When the companies determine their own

outputs, they both naively believe that the competitor will not change production quantities and will pursue the goal of profit maximization, though the market price of the product is still determined based on the combined outputs of both companies. The Bertrand model assumes that two companies in a duopolistic market produce homogeneous products and face the same demand curve. The company that enters the market first sets the price according to its production capacities and profit maximization goals. The second company that enters soon after only needs to set a slightly lower price and waits to make a clean sweep of the entire market. As a result, the two companies compete on prices until the profits of both parties are zero. The Stackelberg model assumes that the leading company in a duopolistic market knows that the other company will engage in production according to the Cournot model. As a result, the leading company uses a naïve company output as a given in its own output decisions and then determines its own output level based on the principle of profit maximization.

The aforementioned traditional production models examine enterprise competition and production decisions primarily from the perspective of static analysis or comparative static analysis. Primary variables considered include demand, price, and cost. These models also use optimization mathematics to derive a perfect closed solution form under conditions of maximum profit. Though these traditional models are elegant and well formulated, perspectives used in these models and factors under consideration appear to extend beyond one's reach due to processes of dynamic evolution. This paper builds on traditional analysis models by employing dynamic analysis perspectives and by considering the evolution of each variable at different times. Namely, variables considered in the model have been afforded additional fitness via stochastic processes or time series models. Variable dynamic behavior paths can then be further generated via the Monte Carlo simulation method. Finally, companies can determine optimal production allocation status levels for a dynamic environment using the stochastic programming method and can then project expected profit levels. The next section provides a detailed description of the stochastic model setup, estimation, and simulation and the optimization model approach.

MODEL

System Setting

It is assumed that a high-tech company implements just-in-time (JIT)³ manufacturing producing two products: a high-end product and a low-end product. The primary investment limits on these two products are the operating hours of the machinery and equipment. To better conform this based on real phenomena, assume that the demands, prices, and costs of both products are uncertain. Furthermore, the company's production capacity is limited to a certain period. The company may be unable to simultaneously meet demands for the two products, or the production costs of outsourcing may be cheaper than those of in-house production. Therefore, production decisions include two alternative plans for in-house production or outsourcing. However, due to the presence of trade secret factors, only low-end products are outsourced. Thus, as a rational decision-maker, we expect to be able to arrive at the optimal production allocation combination in the preceding scenarios

³ Assumes that this company produces only after receiving an order, and therefore only an inventory system that can be ignored exists in the system (Hutchins, 1999).

using initial profit maximization as a guideline for future production over a certain period.

To obtain the optimal production allocation level, we must first define variables and parameters of the system to facilitate subsequent mathematical formula display, simulation, and solution. We assume that variables in the system include seven uncertainty origins: market demand for the high-end product (Q_H), market demand for the low-end product (Q_L), market price for the high-end product (P_H), market price for the low-end product (P_L), cost of in-house high-end product production (V_H)⁴, cost of in-house low-end product production (V_{L1}), and low-end product outsourcing payment cost (V_{L2}). Furthermore, system parameters include the known fixed number of hours spent on in-house high-end product production (T_H), hours spent on in-house low-end product production (T_{L1}), and the upper limit of company machinery and equipment operating hours (T_E). Finally, proposed model decision variables include the company's in-house high-end product production capacity (Q_H^S), the company's in-house low-end product production capacity (Q_{L1}^S), and the company's outsourced low-end product production capacity (Q_{L2}^S). Based on these, the multiphase optimization problem formula can be written as follows:

First, the mathematical equation for the profit $\pi_H(t)$ at time t of in-house high-end product production is:

$$\pi_H(t) = \max\{[P_H(t) - V_H(t)]Q_H^S(t), 0\} \quad (1)$$

The mathematical equation for the profit $\pi_{L1}(t)$ at time t when the company decides to engage in in-house low-end product production is:

$$\pi_{L1}(t) = \max\{[P_L(t) - V_{L1}(t)]Q_{L1}^S(t), 0\} \quad (2)$$

The mathematical equation for the profit $\pi_{L2}(t)$ at time t when the company decides to engage in outsourced low-end product production is:

$$\pi_{L2}(t) = \max\{[P_L(t) - V_{L2}(t)]Q_{L2}^S(t), 0\} \quad (3)$$

The mathematical equation for the overall profit can be determined by integrating the three aforementioned equations:

$$\pi(t) = \pi_H(t) + \pi_{L1}(t) + \pi_{L2}(t) \quad (4)$$

We can add a related boundary equation and solve for the optimal production allocation decision via stochastic programming:

$$\begin{aligned} \text{Target function equation : } & \max_{\{w_H, w_{L1}, w_{L2}\}} E_0 \left[\sum_{t=0}^T \pi(t) \right] \\ & = E_0 \left\{ \sum_{t=0}^T [\pi_H(t) + \pi_{L1}(t) + \pi_{L2}(t)] \right\} \end{aligned}$$

Boundary equations: $Q_H(t)T_H + Q_{L1}(t)T_{L1} \leq T_E$ [Production resource boundary equation]

$$Q_H^S \leq Q_H(t) \quad \text{[High - end product demand boundary equation]}$$

$$Q_{L1}(t) + Q_{L2}(t) = Q_L(t) \quad \text{[Low - end product demand boundary equation]}$$

$$Q_H^S, Q_{L1}^S, Q_{L2}^S \geq 0 \quad (5)$$

STOCHASTIC VARIABLES

Stochastic Process Models

To portray (adapt) the dynamic behaviors of the seven stochastic variables in the system, we separately assume that Q_H and Q_L obey the jump diffusion process (JDP) and that P_H , P_L , V_H , V_{L1} , and V_{L2} obey geometric Brownian motion (GBM) principles. As

⁴ Assume that this company uses activity-based costing (ABC) and can effectively sum up the cost driver. Therefore, there is no need to differentiate between changing and fixed costs (Cooper and Kaplan, 1991, 1992; Kaplan and Anderson, 2004).

market demands often function under irregular jump conditions, the JDP is used to generate additional accessories. The JDP is an extension of the GBM (please refer to Kou (2002) for setup and estimation specifications).

$Y(t)$ is used below to comprehensively represent the behaviors of the seven stochastic variables listed above at time t . This can be simplified as Y to separately explain the modeling processes of JDP and GBM:

When Y obeys GBM⁵:

$$d\log Y(t) = \left(\mu_Y - \frac{1}{2}\sigma_Y^2\right)dt + \sigma_Y dW_Y \quad (6)$$

μ and σ are the individual stochastic variable average and fluctuation parameter, respectively. These can be estimated through a maximum likelihood estimation (MLE) of each variable's historical data; dW is the Wiener process, $dW = \varepsilon\sqrt{t}$, $\varepsilon \sim N(0,1)$; and dt is the time distance.

When Y obeys JDP:

$$d\log Y(t) = \left(\mu_Y - \frac{1}{2}\sigma_Y^2\right)dt + \sigma_Y dW_Y + GdJ_Y \quad (7)$$

dJ_Y obeys the Poisson process based on parameter λ . When $dt \rightarrow 0$, the probability of $dJ_Y = 1$ is λdt , and the probability of $dJ_Y = 0$ is $1 - \lambda dt$; G is an independent normal stochastic variable with an average of k and a partial variance of s^2 . JDP and GBM mainly differ in that JDP allows the sequence to include a jump (discontinuous) behavior through the externally added GdJ_Y term so that it can better capture the discontinuous characteristics of demand. Redner and Walker (1984) noted that MLE cannot estimate parameters of JDP, and thus, it is necessary to change to the expectation-maximization algorithm (EM).

To conduct a Monte Carlo simulation, we divide time into N equal segments. The duration of each time segment is Δt . Thus, the aforementioned continuous form can be transformed into the following discrete form:

$$\log Y(t + \Delta t) - \log Y(t) = \left(\mu_Y - \frac{1}{2}\sigma_Y^2\right)\Delta t + \sigma_Y \varepsilon \sqrt{\Delta t} \quad (8)$$

$$\log Y(t + \Delta t) - \log Y(t) = \begin{cases} \left(\mu_Y - \frac{1}{2}\sigma_Y^2\right)\Delta t + \sigma_Y \varepsilon \sqrt{\Delta t}, & \text{if } U > \lambda \Delta t \\ \left(\mu_Y - \frac{1}{2}\sigma_Y^2\right)\Delta t + k + \varepsilon \sqrt{\sigma_Y^2 \Delta t + s^2}, & \text{if } U \leq \lambda \Delta t \end{cases} \quad (9)$$

Where $\varepsilon \sim N(0,1)$, $U \sim U(0,1)$, and where ε and U are mutually independent stochastic variables.

Time Series Models

The fit of stochastic variables can be described based on the stochastic process and can be determined using the ARMA-GARCH model. Although the principles of the two methods differ, their results are similar. We can also assume Q_H , Q_L , P_H , P_L , V_H , V_{L1} , and V_{L2} obey the ARMA(R,M)-EGARCH(P,Q) process. $Y(t)$ is used below to comprehensively represent the behaviors of the seven stochastic variables listed above at time t . Namely:

$$Y(t) = c + \sum_{i=1}^R \phi_i Y(t-i) - \sum_{j=1}^M \theta_j a(t-j) + a(t) \quad (10)$$

wherein c is a floating term, ϕ_i is the weight of $Y(t-i)$ during the previous i th period, and θ_j is the weight of innovation $a(t-j)$ during the previous j th period. Equation (10) is the ARMA(R,M) model proposed by Box and Jenkins (1976) for use in capturing the innovation autocorrelation effect of Y at time t with its pre- M and pre- R periods. $a(t)$ obeys the average value of zero, and the variables follow a time-varying $h(t)$ normal allocation process. More specifically, to fit the possible

⁵ The equation can also be written as $Y(t)/Y(t) = \mu_Y dt + \sigma_Y dW_Y$, or $dY(t) = \mu_Y Y(t)dt + \sigma_Y Y(t)dW_Y$.

volatility clustering phenomena⁶ of each stochastic variable, the EGARCH(P,Q)⁷ method proposed by Nelson (1991) is used to fit the behavior of $h(t)$. Its equation is as follows:

$$\ln[h(t)] = k + \sum_{i=1}^P \alpha_i \ln[h(t-i)] + \sum_{j=1}^Q \beta_j [|\tilde{a}(t-j)| - E[|\tilde{a}(t-j)|]] + \sum_{j=1}^Q \gamma_j \tilde{a}(t-j) \quad (11)$$

where $\tilde{a}(t) = a(t)/\sqrt{h(t)}$ is the standardized innovation level at time t , α_i is the weight of the logarithmic condition variable $\ln[h(t)]$ during the previous i th period, and β_j is the weight of standardized innovation $\tilde{a}(t-j)$ during the previous j th period. Furthermore, γ_j allows the effects of changes during the conditions to have an asymmetric effect on positive or negative innovation. When $\gamma_j < 0$, negative innovation lag $\tilde{a}(t-j)$ will cause higher conditional heterogeneous variations relative to positive innovation lag levels and vice versa.

The company's actual data can be used to estimate the parameters in ARMA(R,M)-EGARCH(P,Q) by gradually increasing the order of R , M , P , and Q and using maximum likelihood estimation to jointly estimate model parameters ϕ_i , θ_j , α_i , β_j , and γ_j . In addition, use sample autocorrelation functions (ACFs), sample partial autocorrelation functions (PACFs), Bayesian information criterion (BIC), and Ljung-Box portmanteau test together to determine the optimal order.

CONCLUSION AND RECOMMENDATION

Conclusion

This paper introduced stochastic process models and a time series model for measuring production decision-making variables and proposed a stochastic programming model that can determine optimal multiphase in-house production or outsourcing quantities. Decision-makers can establish parameters of system modeling by estimating historical data or by making subjective judgments of trends. Then, to go one step further, this approach combined with Monte Carlo simulation can generate multiple dynamic routes that can measure the variables. In the end, by substituting these routes into the stochastic optimization model proposed in this paper, optimal quantities of in-house production or outsourcing can be obtained.

As the high-tech industry has faced extremely uncertain demands since the end of the 20th century, traditional static and comparative static production decision models can no longer effectively serve as a basis for production decisions on allocation. Therefore, the model proposed in this paper can be used to improve the quality of high-tech company decision-making by allowing for the determination of production allocation levels and can serve as the basis for future financial forecasts.

Recommendation

Extensions of this research may add matrix decomposition mathematical principles (e.g., Cholesky decomposition or singular value decomposition (SVD)) to the methods described here and may further transform the stochastic variables that decisions must address into linked stochastic variables to better reflect conditions of practical decision making.

⁶ This shows that the fluctuation rate is correlated. This is attributable to simultaneously occurring major and minor fluctuations.

⁷ EGARCH can directly ensure that the estimated variable results are greater than zero and can capture asymmetric impact outcomes. For example: the effect of bad news on fluctuations in the status variable may be greater than the effect of good news.

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