A Spreadsheet Optimization Model for Solving Sudoku Problems
Daniel D. Friesen¹, Mike C. Patterson² and Bob Harmel³

Abstract
Sudoku is an extremely popular number puzzle whose popularity has been translated into significant economic activity as well as earning a spot in the existing popular culture. The model presented in this paper uses modern spreadsheet optimization methods to solve the puzzle. This contemporary optimization approach demonstrates an interesting and powerful application of spreadsheet add in software. Given the popularity of Sudoku puzzles, it may be of significant value in teaching and learning operations research and management science.

Key words: Spreadsheet; Sudoku; Optimization Model; Puzzle

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INTRODUCTION
Sudoku is an extremely popular number puzzle that millions of people visit regularly, often during the course of reading their preferred print media such as a daily newspaper or news magazine. Other sources include periodicals devoted to the puzzle, mobile telephone games, handheld video game consoles, and desktop personal computers—via both locally-stored and on-line puzzle generators. Sudoku is the subject of live television shows and competitions (Wikipedia, n.d.). The publisher of Sudoku Selection magazine reported expected sales of 850,000 per issue (Management Today, 2005). There is a board game manufactured by BriarPatch, Inc. Gould supplies free Sudoku puzzles to 400 newspapers in 60 countries through his company, Pappocom (Torres, 2006). Shortz (2006) reports that the puzzle is ubiquitous, appearing in “nearly every major newspaper in America.” Sudokushop.com and Ilovesudoku.com offer a host of Sudoku accessories, e.g. calendars, mouse pads, t-shirts, etc. (Torres, 2006). A recent listing of E-bay’s products yields over 1500 hits for the keyword Sudoku. It has replaced crossword puzzles in several UK newspapers (Strategic Finance, 2005). Outgrowths of Sudoku include “Wordoku” which uses letters instead of numbers (wordoku.biz). As a part of popular culture, the puzzle provides an excellent opportunity to discuss or consider solution algorithms and methods at a higher-than-typical interest level. That is, as an optimization problem, Sudoku is a lot less abstract than some of the typically encountered pedagogical problems. In this paper, we develop and report on an optimization model for solving Sudoku puzzles. In the Literature Review, we discuss the history, nature of the problem behind the puzzle, and solution approaches. Sudoku puzzles require no mathematics to solve by hand; however, the puzzle “poses a number of intriguing mathematical problems” (Delahaye, 2006).

LITERATURE REVIEW - HISTORY
Sudoku™ was invented in 1979 by Howard Garns, an American. Dell Magazines published the first puzzle using the name “Number Place” (Sudoku History, n.d.). The puzzle acquired the name “Sudoku” in Japan in 1986. “Sudoku” is an abbreviated version of a longer phrase that, in Japanese, means “the digits must occur only once.” In 1997, Wayne Gould discovered a Sudoku puzzle book in a Tokyo bookstore. He became so captivated by the puzzle that he coded a computer program to generate puzzles. Subsequently, he convinced the London Times to offer a daily puzzle (Campbell, 2005). The popularity of Sudoku has increased significantly over the past ten years.

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STRUCTURE
Delahaye (2006) describes the structure of Sudoku:

“Typically it contains 81 cells (nine rows and none columns) and is divided into nine smaller squares containing nine cells each; call them subgrids. The game begins with numbers already printed in some cells. The player must fill in the empty cells with the numbers 1 to 9 in such a way that no digit appears twice in the same row, column or subgrid. Each puzzle has one unique solution.”

While the solution may be elusive, verifying the correctness of the solution is easy (Campbell, 2005), which may help explain the popularity.

Delahaye (2006) is referring to the standard puzzle; other variations exist, some more difficult (including a massive 25 cell by 25 cell grid) and some less so. In May 2005, Dion Church published a three-dimensional Sudoku puzzle (Suchard, Yatom, and Shapir, 2006). Solving a puzzle by hand is an exercise in logic as opposed to mathematics (Campbell, 2005; Delahaye, 2006). The numbers could be replaced with distinct symbols, letters, or colors with no impact on puzzle rules. ESPN has substituted baseball team positions for the numbers (Wikipedia, 2007).

Sudoku puzzles vary in difficulty depending on the positions and “relevance” of preprinted numbers, as opposed to the number of preprinted numbers given. In fact, difficulty is not necessarily proportional to the number of givens. Sudoku puzzles are generally coded with a difficulty level. Labels for these difficulty levels vary among publications but are typically synonymous with the following terms: easy, intermediate, hard, and challenging. Levels may be determined by test solvers, either human or machine (Wikipedia).

Sudoku belongs to an important class of mathematical problems known as NP-complete. It has been shown that the number of unique completed grids is 5,472,730,538. This number excludes grids that can be derived from other grids by using elementary operations, e.g. starting with a solution grid and adding 1 to each number with nines becoming ones. If one does not exclude equivalent configurations, the number of possible solution grids explodes to approximately 6.67E21. Any given complete grid can be solved from multiple starting grids; to date, no one has determined the number of unique starting grids. So far, the minimum number of clues that result in a unique solution is thought to be 17. The maximum number of clues that results in a non-unique solution is 77 (Delahaye, 2006).

SOLUTION METHODS
Humans and computers tend to use different solution approaches. A three component solution strategy used by human puzzle solvers consists of some combination of (1) scanning, (2) marking up, and (3) analyzing. Scanning involves systematic examination of rows, columns, and subgrids for the requisite complement of nine digits. Marking up involves labeling cells either with possible or not-possible answers. Analysis either eliminates enough possible numbers to leave only one possible answer for a given cell, or institutes a guess and places digits in their logical conclusion as determined by the guess. Eventually, analysis allows a conclusion about the guess (either correct or not) (Wikipedia, 2007).

The most common computing solution uses an approach called backtracking. Delahaye (2006) describes backtracking as a systematic form of trial and error in which partial solutions are proposed and then modified slightly as soon as they are proved wrong. ... In a well-written program, this method exhaustively explores all possible hypotheses and thus finishes by finding the solution, if one exists. ... Backtracking techniques can be encoded by fairly short solution programs.


OPTIMIZATION APPROACH WITH SPREADSHEET IMPLEMENTATION
In this paper, we demonstrate the solution of a Sudoku puzzle—an NP-complete combinatorial problem—through a spreadsheet implementation. Rather than using specialized optimization software or a general programming language to perform the optimization, we formulate an Excel spreadsheet
solution that utilizes Excel add-in optimizing tools. This solution approach conforms to the changing emphasis encountered in teaching the fields of operations research and management science. According to Meredith, Shafer, and Turban (2002), optimization is one of the most widely taught topics in the management science discipline. However, the subject of optimization has fundamentally shifted emphases over time. Interpreting and understanding the results have become the predominant foci while performing “tedious” calculations and mastering seemingly arcane programming software are no longer emphasized or expected. The shifting emphasis has made spreadsheet software, especially Microsoft Excel, an important optimization tool (Meredith, Shafer, and Turban, 2002). Solver, developed by Frontline Systems and included as an add-in tool with Microsoft Excel, possesses computing capability for small- to moderately-sized models.

Unfortunately, Solver proved incapable of solving the problem proposed in this paper. Premium Solver Platform, also developed by Frontline Systems, proved capable. Premium Solver Platform uses an evolutionary optimization engine based on genetic algorithms.

SUDOKU MODEL

The problem selected to illustrate the spreadsheet Sudoku model is displayed in Figure 1.

**Figure 1**

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There are many Sudoku web sites available. This problem was selected from http://www.websudoku.com (N.D.). The web sites rates this problem as “evil” meaning it is a very difficult or challenging problem. A general statement of the Sudoku problem optimization formulation is shown below. Figure 2 presents the problem in the standard Sudoku 3X3 matrix.

1. Each row must contain integers between 1 and 9 and each integer can occur in each row only once.
2. Each column must contain integers between 1 and 9 and each integer can occur in each column only once.
3. Divide the 9 by 9 matrix into 9 nine 3 by 3 subgrids, as displayed in Figure 2. Each of the nine sub grids must contain integers between 1 and 9 and each integer can occur only once.
SPREADSHEET MODEL FORMULATION
The software used to develop the model is Premium Solver Platform, Version 7, which was released by Frontline Systems in mid 2006. Frontline Systems is the software company that developed the standard Solver Excel add-in tool. The standard add-in does not have a feature that the Sudoku model required. The feature that is utilized is the “alldifferent” constraint and is available in the Premium Solver Platform. The Frontline Systems web site (solver.com) is a good source for information on optimization. The initial spreadsheet formulation is displayed in Figure 3. The matrix in cells B3:J11 is the initial problem. The three matrices displayed in cells B13:J21, B23:J31 and B33:J41 will hold the final solution. Three matrices are required since the “alldifferent” constraint does not allow for the mixing of row and column values. Specifically, one cannot say that the values in a row and a column in the same matrix use the “alldifferent” constraint. The matrix in cells B13:J21 will require that row values will be all different. The matrix in cells B23:J31 will require that column values are all different. Likewise, the matrix in cells B33:J41 will assure that the values in each of the 3 by 3 sub grids (Figure 2) will be all different. One other requirement is that the beginning values displayed in Figure 1 do not change. This will be ensured with constraints that reference formulas in the cells L3:T11. The formula view of these cells is displayed in Figure 4. The resulting values in these cells will be required to be equal to zero. If the initial values are equal to zero, then the value in the corresponding matrix will be set to zero. However, if the initial value is not equal to zero, a constraint will require that the initial value be moved to the solution matrix. The Premium Solver Platform parameters are displayed in Figure 5. The “Set Cell” value is left blank, since there is not a single cell value that the model is attempting to optimize. The “Changing Variable Cells” refer to the three solution matrices (cells B13:J21, B23:J31 and B33:J41). The solver engine selected is “StandardLP/Quadratic”. The “Subject to the Constraints” section holds the constraints. The “alldifferent” constraints assure the solution matrices (column, rows and 3X3 sub-matrices) contain an integer value only once. The two “=” constraints, B13:J21=B23:J31 and B23:J31=B33:J41, assure that the three solution matrices are identical. The = zero constraint, L3:T11=0, assures that the non-zero initial values are located in the solution matrix. The final solution is displayed in Figure 6. The display of all three solution matrices, which hold identical values could be viewed as redundant. For display purposes, one might want to hide the final two matrices located in cells B23:J31 and B33:J41.

SUMMARY
Sudoku is an extremely popular number puzzle whose popularity has been translated into significant economic activity as well as earning a spot in the existing popular culture. The model presented in this paper uses modern spreadsheet optimization methods to solve the puzzle. This contemporary optimization approach demonstrates an interesting and powerful application of spreadsheet add in software. Given the popularity of Sudoku puzzles, it may be of significant value in teaching and learning operations research and management science.

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Figure 3
Initial Spreadsheet Formulation of Sudoku Model

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Figure 4
Formula View of Sudoku Model
(Cells L3:T11)

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3  =IF(B3=0,0,B13-B3) =IF(C3=0,0,C13-C3) =IF(D3=0,0,D13-D3) =IF(E3=0,0,E13-E3) =IF(F3=0,0,F13-F3)
4  =IF(B4=0,0,B14-B4) =IF(C4=0,0,C14-C4) =IF(D4=0,0,D14-D4) =IF(E4=0,0,E14-E4) =IF(F4=0,0,F14-F4)
5  =IF(B5=0,0,B15-B5) =IF(C5=0,0,C15-C5) =IF(D5=0,0,D15-D5) =IF(E5=0,0,E15-E5) =IF(F5=0,0,F15-F5)
6  =IF(B6=0,0,B16-B6) =IF(C6=0,0,C16-C6) =IF(D6=0,0,D16-D6) =IF(E6=0,0,E16-E6) =IF(F6=0,0,F16-F6)
7  =IF(B7=0,0,B17-B7) =IF(C7=0,0,C17-C7) =IF(D7=0,0,D17-D7) =IF(E7=0,0,E17-E7) =IF(F7=0,0,F17-F7)
8  =IF(B8=0,0,B18-B8) =IF(C8=0,0,C18-C8) =IF(D8=0,0,D18-D8) =IF(E8=0,0,E18-E8) =IF(F8=0,0,F18-F8)
9  =IF(B9=0,0,B19-B9) =IF(C9=0,0,C19-C9) =IF(D9=0,0,D19-D9) =IF(E9=0,0,E19-E9) =IF(F9=0,0,F19-F9)
10 =IF(B10=0,0,B20-B10) =IF(C10=0,0,C20-C10) =IF(D10=0,0,D20-D10) =IF(E10=0,0,E20-E10) =IF(F10=0,0,F20-F10)
11 =IF(B11=0,0,B21-B11) =IF(C11=0,0,C21-C11) =IF(D11=0,0,D21-D11) =IF(E11=0,0,E21-E11) =IF(F11=0,0,F21-F11)

Figure 5
Premium Solver Platform Parameters

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## Figure 6
### Final Solution to Sudoku Model

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