

## Polynomial Distribution of Market Share

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### Abstract

*Zipf distribution is known to describe various natural and social phenomena, including size of starts, city populations, words frequency in literature, immune systems, and size of firms. In this paper we show that market shares in the world cola-drink market follows a special kind of skewed distribution, which we call polynomial distribution*

**Key words:** Polynomial Distribution of Market Share, Zipf distribution, market share.



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### INTRODUCTION

For many decades, economists have paid a lot of attention to size distribution of business firms. However, this issue—albeit its importance for practitioners—has been of little interest to management scholars. This has also been the case with market share. Although many companies have adopted formalized approaches to strategic planning in which market share is a primary determinant of costs and profitability, in the management and marketing fields (with the exception of Buzzell (1981) who found that semi-logarithmic distribution of market share in narrow-defined product markets is so common that it can be considered a “natural” phenomenon), market share distribution has been practically overlooked. In this paper we show that the distribution of market shares in the world cola-drink industry are polynomial-distributed. This empirical finding poses a tremendous challenge to conventional wisdom and theories and reveals the need to build compelling explanations for how performance differences among firms arise. The remainder of this paper is organized as follows. The next section provides a brief description of the origins and characteristics of Zipf distribution. Section three is a review of literature about the distribution of market share. Section four describes the test data set and the statistical methodology. The paper finishes with some conclusions.

### SELF-ORGANIZATION AND ZIPF DISTRIBUTION

In self-organizing systems, initially, there are unnoticeable differences, but over time those small differences become magnified through a process of self-reinforcement, which produce large-scale skewed distributed patterns. The most apparent characteristic of self-organizing systems is that their size distribution usually follows Zipf’s Law. This law was introduced by the Harvard University linguistic Professor George Kingsley (1945, 1949), who asserted that the relationship between the size of data values and their rank approximately follows the law:

$$S_r r = k \quad (1)$$

Where  $S_r$  is the element of the set of data ranked  $r$  in a list of elements ordered by size, beginning by the largest.

To visualize how Zipf’s law behaves, we take the elements of a data set and order them by size. We then draw a graph; on the x-axis we place the natural log of the size,  $S_r$ , and on the y-axis the natural log of the rank,  $r$ , of the corresponding element ( $s_1 > s_2 > \dots > s_N$ ). If we run a regression, we obtain:

$$\ln r = \beta_1 + \beta_2 \ln S_r + \varepsilon_1 \quad (2)$$

If the slope of the regression line ( $\beta_2$ ) is close to  $-1$ , then this data set is said to follow a Zipf distribution. If the slope of the regression line ( $\beta_2$ ) is different from  $-1$ , it may follow a generalized Zipf distribution:

$$S_r r^q = k \quad (3)$$

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The Zipf-Law has been used to describe the distribution of words in a variety of languages (Zipf, 1945), the size of cities (Gabaix, 1999), immune system response (Burgos and Moreno-Tovar, 1996; Li, 2001), and size of firms (Axtell, 2001).

### ZIPF DISTRIBUTION OF MARKET SHARE

The firm size distribution within an industry is a relevant performance measure, for it indicates the degree of industrial concentration, a quantity of particular interest for antitrust policy. However, management practitioners may be interested in more specific ways to measure firms' performance. One of these ways is related to market share. Reimer et al (2002), in their study about market share distribution, report that the top 25 retailers sell more than half (53%) the volume generated by manufacturers like Kraft and General Mills. The balance is sold through thousands of other retailers. Concerning the Consumer Packaged Goods (CPG) market, the top 10 retailers had a 43% share in the year 2000, while the top 11-25 retailers had a total of only 10% market share. Top two manufacturers of personal computers (HP and Dell) held about 30.4% share of US personal computer market for the second quarter of 2002, while the rest was divided among many different firms (IBM was the third largest manufacturer of personal computer with only 6.6% market share). From these specific cases, it is not far-fetched to deduce that market shares within a market tend to be highly skewed.

Furthermore, Reimer et al (2002), using data from 70 markets, argue that market shares within these markets are Zipf distributed. We can infer from this assertion that, if we take the market share of the firms of a given industry and rank them according to their magnitude and we plot these magnitudes against their rank using a logarithmic scale, the slope of the resulting line is close to -1. Although these findings are a compelling evidence of what any accurate marketing theory has to explain, in the next section we show that there is no reason to expect the distribution of market shares to take any particular form for the general run of *industries*.

### POLYNOMIAL DISTRIBUTION OF MARKET SHARE IN THE WORLD COLA-DRINK INDUSTRY

Empirical investigation –as we saw in the previous section– has shown that distribution of market shares in different industries are Zipf-distributed. However, in this section we prove that distribution of market share may vary in systematic way from one industry to another. Using market share data from the world cola soft-drink industry in 2009 (see Appendix A), we show that Zipf distribution does not describe the market share of the 28 most important brand names participating in this industry. If we regress the log of volume on the log of rank (see values Appendix A) of the 28 brand names, we get a  $R^2$  of 0.9513 and a slope of -0.5323. This result is not close to -1, so it does not follow Zipf's law.

*Insert figure 1 here*

A more precise description of the distribution of market share can be obtained by finding what we can call the *polynomial distribution of rank*. As Figure 1 shows, the rank-market share relationship is nearly cubic in the logs and close to the equation:

$$\ln R = -0.03(\ln MS)^3 + 0.6989(\ln MS)^2 - 5.7651 \ln MS + 18.27 \quad (4)$$

Here  $R^2 = 0.9724$ , which means that this equation fits better the data than the linear regression.

If we calculate the derivative of (4), we obtain the slope of the curve for any value of  $r$ :

$$(\ln R)' = -0.09 (\ln MS)^2 + 1.3978 (\ln S) - 5.7651 \quad (5)$$

Table 1 shows different values of equation (5) for different values of market share (measured in terms of million liters). That the slope is -1.03 implies that the first brand name has a much large market share than the second one. However, the difference between the second brand name's market share and that of the third brand name is lower. That is why the slope is -0.5 and -0.37. Of course, market share distribution in the cola soft-drink industry does not follow Zipf's law, it is clear that it is still a very polarized distribution.

*Insert table 1 here*

### CONCLUSIONS

Few analyses have addressed the question of why markets follow a polynomial-distribution of market share. In this paper we have found an especial kind of distribution which can be called polynomial distribution. We showed that this especial kind of distribution fit very well the behavior of market share

of the cola soft-drink industry. This is an obvious target that any accurate marketing and economic theory should hit.

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## Appendix A

### Market share by global brand name of the world soft-drink industry

Brand names	Company name (GBO)	Market share	Volume (Mn Liters)	Ln of rank	Ln of volume
Coca-Cola	Coca-Cola Co, The	24,8	37842	0	10,54
Pepsi	PepsiCo Inc	10,8	16479	0,69	9,71
Diet Coke	Coca-Cola Co, The	5,6	8545	1,10	9,05
Diet Pepsi	PepsiCo Inc	2,4	3662	1,39	8,21
Coca-Cola Zero	Coca-Cola Co, The	1,8	2747	1,61	7,92
Big Cola	AJEGROUP	1	1526	1,79	7,33
Pepsi Max	PepsiCo Inc	0,5	763	1,95	6,64
Pepsi Twist	PepsiCo Inc	0,3	458	2,08	6.13
RC Cola	Cott Corp	0,3	458	2,20	6.13
Pop Cola	Coca-Cola Co, The	0,2	305	2,30	5,72
Future	Hangzhou Wahaha Group	0,2	305	2,40	5,72
Cherry Coke	Coca-Cola Co, The	0,2	305	2,48	5,72
Zamzam	Zamzam Beverage Co	0,2	305	2,56	5,72
Inca Kola	Coca-Cola Co, The	0,2	305	2,64	5,72
Kola Real	AJEGROUP	0,2	305	2,71	5,72
Caffeine Free Coca-Cola	Coca-Cola Co, The	0,2	305	2,77	5,72
Tab	Coca-Cola Co, The	0,2	305	2,83	5,72
Barg’s	Coca-Cola Co, The	0,1	153	2,89	5,03
Sinalco	Sinalco International GmbH & Co KG	0,1	153	2,94	5,03
Thums Up	Coca-Cola Co, The	0,1	153	3,00	5,03
Hoop	Kofola SA	0,1	153	3,04	5,03
La Casera	Suntory Holdings Ltd	0,1	153	3,09	5,03
Cola Turka	Ülker Gıda Sanayi ve Ticaret AS	0,1	153	3,14	5,03
Pepsi Wild	PepsiCo Inc	0,1	153	3,18	5,03

Cherry					
Vanilla Coke	Coca-Cola Co, The	0,1	153	3,22	5,03
Mr PiBB	Coca-Cola Co, The	0,1	153	3,26	5,03
Hit	Coca-Cola Co, The	0,1	153	3,30	5,03
Caffeine Free Pepsi	PepsiCo Inc	0,1	153	3,33	5,03
Others			8644		
Total			85548,4		

Figure 1 Distribution of ln (volume) in mn liters vs. ln (rank) in the cola soft-drink industry

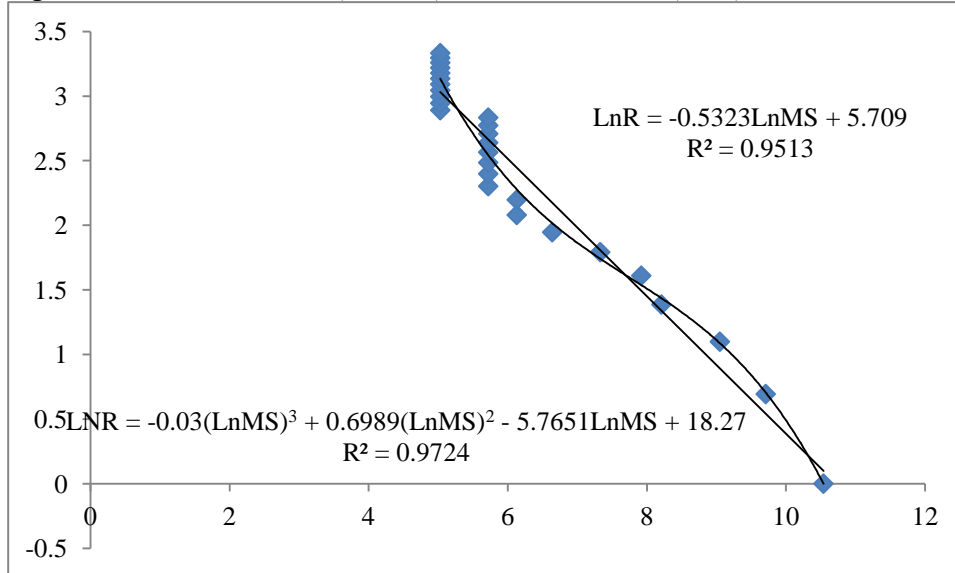


Table 1 different values of equation (5) for different values of S

S(Volume) (Mn liters)	ln Volume	Slope
37841	10,54	-1,03
8544	9,05	-0,50
3662	8,21	-0,37
1525	7,33	-0,37
762	6,64	-0,47
456	6,12	-0,60
309	5,73	-0,73
307	5,73	-0,74
306	5,72	-0,74
304	5,72	-0,74
303	5,71	-0,74
302	5,71	-0,74
301	5,71	-0,74
158	5,06	-1,02
157	5,06	-1,02