Application of Extreme Value Copulas to Palm Oil Prices Analysis
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Abstract
In this paper we study the tail behavior of the palm oil future markets using the Extreme Value Theory and focusing on the dependence structure between the returns on palm oil future price in three palm oil futures markets, namely Malaysian futures markets (KLSE), Dalian Commodity Exchange (DCE) and Singapore Exchange Derivatives Trading Limited (SGX-DT) by using the Extreme Value Copulas. The results demonstrated that the returns on palm oil future price among KLSE and SGX-DT have dependence in extreme, whereas the returns on palm oil future price among KLSE and DCE, SGX-DT and DCE do not have any dependence. The results could be beneficial for any person or company wishing to be engaged in the commerce of trading palm oil.

Key words: Extreme Value Theory, Extreme Value Copulas, Dependence structure, Malaysian futures markets, Dalian Commodity Exchange, Singapore Exchange, Palm oil.

INTRODUCTION
Extreme Value Theory (EVT) is a concept that is concerned with the analysis and modeling of extreme high or low observations. The EVT distributed assumption gives the results for the distribution of the normalized maximum of a high number of observations, or equivalently, the distribution of exceedances of observations over a high threshold (Rakonczai and Tajvidi, 2010). Under EVT assumptions on the underlying distribution of observations, it is often superior to normal distribution in many situations and has been widely used in many fields such as financial, hydrological, insurance and environmental science (Lu et al., 2008). The joint extreme events can have some serious impact on a particular field of study; therefore it needs to be carefully modeled. With a calculation of the probability that there is an observation exceeding a certain benchmark, it requires knowledge of the joint distribution of maximal heights during the forecasting period. This is a typical field of application for EVT (Gudendorf and Segers, 2009).

Copulas method has become rapidly developed and has brought the attention in various fields as a way to overcome the limitations of classical dependence measures as exemplified by the linear correlation. The copulas approach is a statistical tool that is considered as the most general margin-free description of the dependence structure of a multivariate distribution (Segers, 2005). The fact that the theory of multivariate maxima in EVT can be expressed in terms of copulas, its philosophy has been recently acknowledged as a form for application. Copulas is revealed to be a very strong tool in financial risk modeling that deals with different classes of existing risks ( Cherubini et al., 2004). Scholars that have implemented the extreme value copulas in their study includes Starica (1999) who had investigated the joint behavior of extreme returns in a foreign exchange rate market, and Lu, Tian and Zhang (2008) who had repeatedly taken up the foreign exchange to analyze the dependence structure between the asset return. The results showed that three copulas are suitable to measure the joint tail risk and tail dependence for markets data. In addition, Longin and Solnik (2001) used EVT to study the dependence structure of international equity markets characterized. An application to the Society of Actuaries medical large claims that the data, in terms of insurance through extreme-value copulas, is the topic of the monograph by Cebrian, Denuit and Lambert (2003)

Palm oil is one of the most important energy-crop in the world (USDA, 2011), its implication as an energy crop is due to being a highly efficient and high yielding source of food and fuel. Palm oil is produced entirely in developing countries. Southeast Asian countries are the largest producing region; palm oil was produced 13.01 million tons in 1992, which increased to 50.26 million tons in 2011, a 286% increase in 19 years (USDA, 2011). Malaysia is one of the world’s biggest palm oil producers. The factors involved in

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setting palm oil prices are quite interesting. According to the relevance of Malaysia’s palm oil price to the Chinese and Singapore markets, it is important to examine the relationship between the Malaysian futures markets (KLSE) and two palm oil futures markets, namely Dalian Commodity Exchange (DCE) and Singapore Exchange Derivatives Trading Limited (SGX-DT). In this paper, we will deal with the tail behavior of the palm oil future markets using the EVT and focusing on the dependence structure between the returns on palm oil future price in three palm oil futures markets, namely KLSE, DCE and SGX-DT by using the extreme value copulas.

The remainder of the paper is organized as followed: Section 2 presents the univariate EVT and Generalized Extreme Value (GEV) distribution, Section 3 reviews the concept of copulas and extreme value copulas. Section 4 explains the data used in the empirical analysis, Section 5 discusses the empirical results, and finally Section 6 offers a conclusion.

UNIVARIATE EVT AND GEV DISTRIBUTION
The main idea of Extreme Value Theory (EVT) is the concept of modeling and measuring extreme events which occur with a very small probability (Brodin and Kluppelberg, 2008). It provides methods for quantifying such events and their consequences statistically. Generally, there are two principal approaches to identifying extremes in real data. The Block Maxima (BM) and Peaks-Over-Threshold (POT) are central for the statistical analysis of maxima or minima and of exceedances over a higher or lower threshold (Lai and Wu, 2007). The BM studies the statistical behavior of the largest or the smallest value in a sequence of independent random variables (Lei and Qiao, 2010; Lei et al., 2011). The POT approach is based on the Generalized Pareto Distribution (GPD) introduced by Pickands (1975) (cited in Lei and Qiao, 2010). These are models for all large observations that exceed a high threshold. In this paper, we will adopt GEV model of the BM method to study the tail behavior of the tail of palm oil future markets.

let \( Z_i \) (i=1,...,n) denote maximum observation in each block. \( Z_n \) is normalized to obtain a non-degenerated limiting distribution. The BM is closely associated with the use of Generalized Extreme Value (GEV) distribution with c.d.f:

\[
H(z) = \exp \left\{ - \left( 1 + \frac{1}{\xi} \left( \frac{z - \mu}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\}
\]

(1)

Where \( \mu, \sigma > 0 \) and \( \xi \) are location, scale and shape parameter respectively. Note that \( \xi > 0 \) is called Frechet distribution, \( \xi < 0 \) is called Fisher-Tippet or Weibull distribution and \( \xi = 0 \) is called Gumble or double-exponential distribution. Under the assumption that \( Z_i, ..., Z_n \) are independent variables having the GEV distribution, the log-likelihood for the GEV parameters when \( \xi \neq 0 \) is given by:

\[
\ell(\xi, \mu, \sigma) = -n \log(\sigma) -\left(1+1/\xi\right) \sum_{i=1}^{n} \log \left[ 1 + \frac{1}{\xi} \left( \frac{Z_i - \mu}{\sigma} \right) \right] - \sum_{i=1}^{n} \left[ 1 + \frac{1}{\xi} \left( \frac{Z_i - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}}
\]

(2)

Provided that \( 1 + \xi \left( \frac{Z_i - \mu}{\sigma} \right) > 0 \), for i=1,...,n

The case \( \xi = 0 \) requires separate treatment using the Gumbel limit of the GEV distribution. The log-likelihood in that case is:

\[
\ell(\mu, \sigma) = -n \log(\sigma) - \sum_{i=1}^{n} \exp \left\{ - \left( \frac{Z_i - \mu}{\sigma} \right) \right\}
\]

(3)

The maximization of this equation with respect to the parameter vector \( (\mu, \sigma, \xi) \) leads to the maximum likelihood estimate with respect to the entire GEV family (see Coles 2001 for detail)

COPULAS AND EXTREME VALUE COPULAS
Copulas have become the attention multivariate modeling in various fields. A copula is a function that links together univariate distribution functions to from a multivariate distribution function (Patton, 2007).
The relevance of copulas stems from a famous result by Sklar (1959) (cited in Segers, 2005). For simplicity, we confined it to the bivariate case. Let X and Y be the stochastic behavior of two random variables with respective marginal cdf’s F(x) and G(y) is appropriately described with joint distribution function
\[ H(x,y) = P(X \leq x, Y \leq y) \]  (4)

And marginal distribution functions
\[ F(x) = P(X \leq x), \quad G(y) = P(Y \leq y) \]  (5)

Since F(x) and G(y) are uniformly distributed between 0 and 1, then the joint distribution function C on [0,1]^2 for all (x,y) ∈ R^2 such that:
\[ H(x,y) = C(F(x), G(y)) \]  (6)

Where C is called the copula associated with X and Y which couples the joint distribution H with it margins. Equation (6) is equivalent to H(F^(-1)(u),G^(-1)(v)) = C(u,v) as a consequence of the Sklar’s Theorem, where u = F(x), v = G(y) are marginal distributions of X,Y. The implication of the Sklar’s Theorem is that, after standardizing the effects of margins, the dependence between X and Y is fully described by the copula (Lu et al, 2008). A comprehensive overview of the copulas properties can be referred to the work by Nelsen (1999). In this paper, we combine the copula construction with the extreme value theory.

The extreme value copula family is used to represent the Multivariate Extreme Value Distribution (MEVD) by the uniformly distributed margins. Consider a bivariate sample (X_i,Y_i), i=1,...,n. Denote component-wise maxima by M_i = max(X_i,...,X_n) and N_i = max(Y_i,...,Y_n). The object of interest is the vector of component-wise block maxima: \( \text{M}_i = (\text{M}_{i1}, \text{N}_{i1}) \). The bivariate extreme distribution H can be connected by an extreme value copula (EV copula) \( C_o \) (Segers, 2005)
\[ H(x,y) = C_o(F(x; \mu_1, \sigma_1, \xi_1), G(y; \mu_2, \sigma_2, \xi_2)) \]  (7)

Where \( \mu_i, \sigma_i, \xi_i \) are GEV parameters and F(x) and G(y) are GEV margin. By Sklar’s Theorem, the unique copula \( C_o \) of H is given by
\[ C_o(u', v') = C_o(u, v), t > 0 \]  (8)

The EV copula has more family. In this paper, the two family applied are Gumbel and Husler-Riess. (Cited in Lu et al., 2008)

Gumbel copula:
\[ C(u, v) = \exp\left(-\left[\ln(-u) + \ln(-v)\right]^\frac{1}{r}\right) \]  (9)

The independence copula is obtained in the limit as r = 1, and complete dependence is obtained in the limit as r = \( \infty \).

Husler-Reiss copula:
\[ C(u, v) = \exp\left\{-u \Phi\left(\frac{1}{r} + \frac{1}{2} r \ln\left(\frac{u}{v}\right)\right) - v \Phi\left(\frac{1}{r} + \frac{1}{2} r \ln\left(\frac{v}{u}\right)\right)\right\} \]  (10)

Where \( \tilde{u} = -\ln u, \tilde{v} = -\ln v \) and \( \Phi \) is the standardized normal distribution. The independence copula is obtained in the limit as r = 0, and complete dependence is obtained in the limit as r = \( \infty \). For the estimation of copulas parameters, we used Exact Maximum Likelihood method (EML): the parameters for margins and copula are estimated simultaneously (see Yan 2007 for details).

**DATA**

This paper used the times series data from DataStream. We work with daily future prices of palm oil data in three markets, namely the Malaysian future markets (KLSE), Dalian Commodity Exchange (DCE) and Singapore Exchange Derivatives Trading Limited (SGX-DT). We took the daily market prices and converted it to a return series. Daily prices are computed as return of market i at time t relative:
\[ R_{i,t} = \ln(p_{i,t} / p_{i,t-1}) \times 100 \] , where \( p_{i,t} \) and \( p_{i,t-1} \) are the daily price of futures for days t and t-1,
respectively. The study period was from December 2007 till June 2012. We have 1196 observations for each market.

**EMPIRICAL RESULTS**

**The parameter estimation of the GEV model**

In the GEV model, we focused on the statistical behavior of block maximum data. Therefore, the source data is set of 55 records of monthly maximum in each market. Table 1 presents the estimation of three parameters of GEV model based on the maximum likelihood method. The results show that the standard error estimates are relatively low. It implies that the block size of data is appropriate for the parameter estimation. Figure 1, 2, 3 presents the scattered plot of the monthly maximum return on KLSE, SGX-DT and DCE, respectively.

**Insert table 1 & figure (1-3) here**

**The parameter estimation of the extreme value copulas**

Table 2 presents the parameter \( r \) estimation in the Gumbel and HuslerReiss copula analysis. In the Gumbel copula method, the parameter \( r \) estimation between KLSE and SGX-DT markets is equal to 3.034, which implies that KLSE and SGX-DT markets have dependence in extreme. Whereas the parameter \( r \) estimation among KLSE and DCE markets, SGX-DT and DCE markets are equal 0.973, 1.065, respectively, thus indicating that KLSE and DCE markets, SGX-DT and DCE markets have neither dependence or even independence in extremes. In the case of HuslerReiss copula, the parameter \( r \) estimation between KLSE and SGX-DT markets is equal to 2.287. This means that KLSE and SGX-DT markets have dependence in extreme, while the parameter \( r \) estimation among KLSE and DCE markets, SGX-DT and DCE markets are equal to 0.220, 0.597, respectively. Thus, there is an indication that KLSE and DCE markets, SGX-DT and DCE markets have neither dependence nor even independence in extremes.

**CONCLUSION**

In this paper, we managed with the tail behavior of return on three palm oil futures prices markets, namely KLSE, DCE and SGX-DT using the univariate EVT and GEV distribution. The study focused on the extreme dependence structure between the returns on palm oil futures prices in three markets using the extreme value copulas. To obtain our results, the paper applied the Gumbel and HuslerReiss copula approach to examine the extreme dependence between KLSE, DCE and SGX-DT markets. The results demonstrated that both methods have a similar outcome. The returns on palm oil future price among KLSE and SGX-DT have dependence in extreme, whereas the returns on palm oil future price among KLSE and DCE, SGX-DT and DCE do not have any dependence. The results could be beneficial for any person or company wishing to be engaged in the commerce of trading palm oil.

**REFERENCES**


**Table 1. The parameter estimation results using the ML method based on GEV model**

<table>
<thead>
<tr>
<th>Market</th>
<th>Parameter estimation</th>
<th>ML Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>KLSE</td>
<td>µ</td>
<td>2.491(0.162)</td>
</tr>
<tr>
<td></td>
<td>σ</td>
<td>1.060(0.135)</td>
</tr>
<tr>
<td></td>
<td>ξ</td>
<td>0.281(0.115)</td>
</tr>
<tr>
<td>SGX-DT</td>
<td>µ</td>
<td>2.736(0.186)</td>
</tr>
<tr>
<td></td>
<td>σ</td>
<td>1.249(0.158)</td>
</tr>
<tr>
<td></td>
<td>ξ</td>
<td>0.319(0.099)</td>
</tr>
<tr>
<td>DCE</td>
<td>µ</td>
<td>2.827(0.333)</td>
</tr>
<tr>
<td></td>
<td>σ</td>
<td>2.186(0.245)</td>
</tr>
<tr>
<td></td>
<td>ξ</td>
<td>0.035(0.104)</td>
</tr>
</tbody>
</table>

*Note*: Terms in parentheses are standard errors of parameter estimates.

**Table 2. Estimation of copula parameter**

<table>
<thead>
<tr>
<th>Market</th>
<th>Gumbel copula</th>
<th>HuslerReiss copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>KLSE-SGX-DT</td>
<td>3.034(0.473)</td>
<td>2.287(0.414)</td>
</tr>
<tr>
<td>KLSE-DCE</td>
<td>0.973(0.084)</td>
<td>0.220(2.721)</td>
</tr>
<tr>
<td>SGX-DT-DCE</td>
<td>1.065(0.079)</td>
<td>0.597(0.156)</td>
</tr>
</tbody>
</table>

*Note*: Terms in parentheses are standard errors of parameter estimates.
Figure 1. The scatter plot of monthly maximum return on KLSE

Figure 2. The scatter plot of monthly maximum return on SGX-DT
Figure 3. The scatter plot of monthly maximum return on DCE